

TRIGONOMETRY 9: QUIZ SOLUTIONS

Question 1

A vertical flagpole, TW , stands with its foot, W , on a horizontal plane. Two observers are at points A and B on the plane. The angle of elevation of the top of the pole, T , from A is α . The distance between the two observers is d and the angle subtended at the foot of the pole, $\angle AWB = \theta$. If $WA = WB$, prove that the height of the

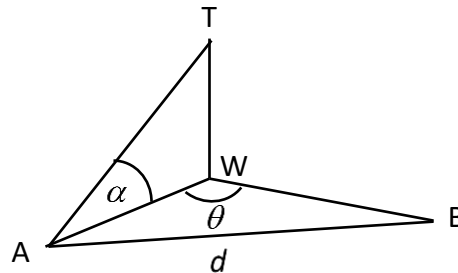
flagpole, h , is given by $h = \frac{d \tan \alpha}{2 \sin\left(\frac{\theta}{2}\right)}$.

Solution

Draw a diagram.

(i) In $\triangle TWA$, $\frac{h}{WA} = \tan \alpha$

$$\therefore WA = \frac{h}{\tan \alpha}$$



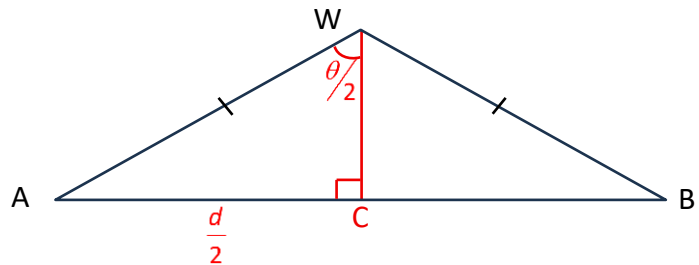
(ii) $\triangle AWB$ is isosceles ($WA = WB$)

Draw perpendicular WC .

$$\therefore \angle AWC = \frac{\theta}{2} \text{ and } AC = \frac{d}{2}$$

$$\frac{\frac{d}{2}}{WA} = \sin\left(\frac{\theta}{2}\right)$$

$$\therefore WA = \frac{d}{2 \sin\left(\frac{\theta}{2}\right)}$$



(iii) So, from (i) and (ii) $\therefore \frac{h}{\tan \alpha} = \frac{d}{2 \sin\left(\frac{\theta}{2}\right)}$

$$\therefore h = \frac{d \tan \alpha}{2 \sin\left(\frac{\theta}{2}\right)}$$

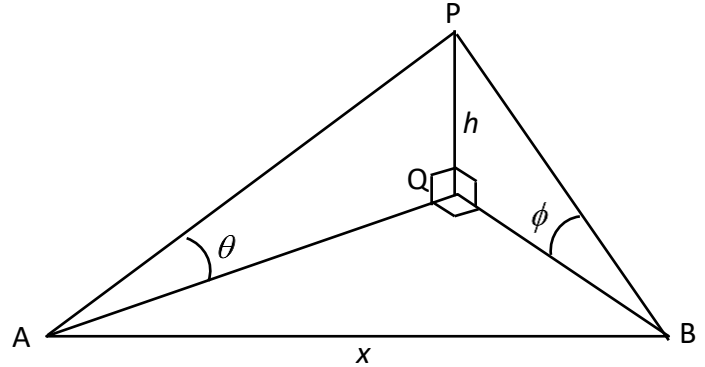
Question 2

A vertical mast PQ of height h stands on a horizontal plane with Q at its base. Two points, A and B, lie on the plane such that the angles of elevation of P from A and B are θ and ϕ , respectively. The distance from A to B is x and $\angle AQB = 90^\circ$.

Show that $x^2 = h^2(\cot^2 \theta + \cot^2 \phi)$

Solution

Draw a diagram.



$$\begin{aligned} \text{(i) In } \triangle PQA, \frac{h}{QA} &= \tan \theta \\ \therefore QA &= \frac{h}{\tan \theta} = h \cot \theta \end{aligned}$$

$$\begin{aligned} \text{(ii) In } \triangle PQB, \frac{h}{QB} &= \tan \phi \\ \therefore QB &= \frac{h}{\tan \phi} = h \cot \phi \end{aligned}$$

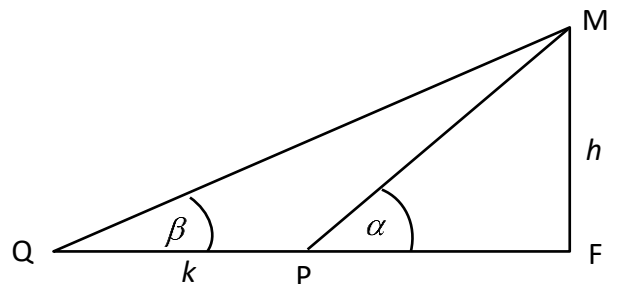
$$\begin{aligned} \text{(iii) In } \triangle AQB, x^2 &= AQ^2 + BQ^2 \\ &= (h \cot \theta)^2 + (h \cot \phi)^2 \\ &= h^2(\cot^2 \theta + \cot^2 \phi) \end{aligned}$$

Question 3

An observer stands at a point P on a horizontal plane and views the top of a mountain, M, at an angle of elevation of α . The observer then walks a distance k directly away from the mountain along the plane to point Q. From Q, the angle of elevation of the top of the mountain is β . If F is the foot of the mountain vertically below M, show that the height, h , of the mountain is given by $h = \frac{k}{\cot \beta - \cot \alpha}$.

Solution

Draw a diagram.



$$\begin{aligned} \text{(i) In } \triangle MFP, \frac{h}{PF} &= \tan \alpha \\ \therefore PF &= \frac{h}{\tan \alpha} = h \cot \alpha \end{aligned}$$

$$\begin{aligned} \text{(ii) In } \triangle MFQ, \frac{h}{QF} &= \tan \beta \\ \therefore QF &= \frac{h}{\tan \beta} = h \cot \beta \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad QP &= QF - PF \\
 \therefore k &= h \cot \beta - h \cot \alpha \\
 &= h(\cot \beta - \cot \alpha)
 \end{aligned}$$

$$\therefore h = \frac{k}{\cot \beta - \cot \alpha}$$

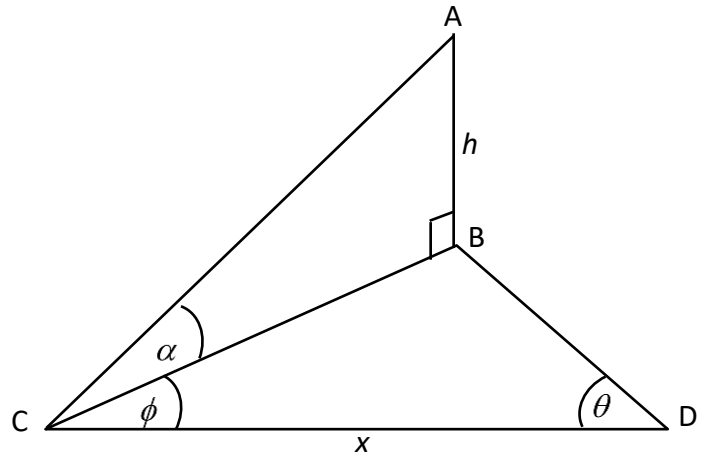
Question 4

A vertical tower AB of height h stands on a horizontal plane. C and D are two points on the plane such that B, C and D form a triangle. The angle of elevation of the top of the tower, A, from C is α . In $\triangle BCD$, $CD = x$, $\angle BDC = \theta$ and $\angle BCD = \phi$.

Show that $h = \frac{x \sin \theta \tan \alpha}{\sin(\theta + \phi)}$.

Solution

Draw a diagram.



$$\text{(i) In } \triangle ABC, \frac{h}{BC} = \tan \alpha$$

$$\therefore BC = \frac{h}{\tan \alpha}$$

$$\text{(ii) In } \triangle BCD, \frac{BC}{\sin \theta} = \frac{CD}{\sin \angle CBD}$$

$$\therefore \frac{BC}{\sin \theta} = \frac{CD}{\sin[180^\circ - (\theta + \phi)]}$$

$$\therefore BC = \frac{x \sin \theta}{\sin(\theta + \phi)}$$

$$\sin(180^\circ - \alpha) = \sin \alpha$$

$$\text{(iii)} \quad \therefore \frac{h}{\tan \alpha} = \frac{x \sin \theta}{\sin(\theta + \phi)}$$

$$\therefore h = \frac{x \sin \theta \tan \alpha}{\sin(\theta + \phi)}$$

Question 5

A vertical communication tower, XY , stands on a horizontal plane with its base at Y . Points A and B are situated on the plane such that $YB = k$. The angle of elevation of the top of the tower, X , from A is θ , $\angle AYB = 90^\circ$ and $\angle YAB = \alpha$. Show that the height of the tower is given by $h = k \cot \alpha \tan \theta$.

Solution

Draw a diagram.

(i) In $\triangle XAY$, $\frac{h}{AY} = \tan \theta$

$$\therefore AY = \frac{h}{\tan \theta}$$

(ii) In $\triangle YAB$, $\frac{k}{AY} = \tan \alpha$

$$\therefore AY = \frac{k}{\tan \alpha}$$

(iii) $\therefore \frac{h}{\tan \theta} = \frac{k}{\tan \alpha}$

$$\therefore h = \frac{k \tan \theta}{\tan \alpha}$$

$$\therefore h = k \cot \alpha \tan \theta$$

