

TRIGONOMETRY 7: QUIZ SOLUTIONS

Question 1

An acute $\triangle ABC$ is inscribed within a circle, centre O , radius r . A diameter is drawn from vertex B through the centre O to meet the circumference at D . A line is drawn from D to C .

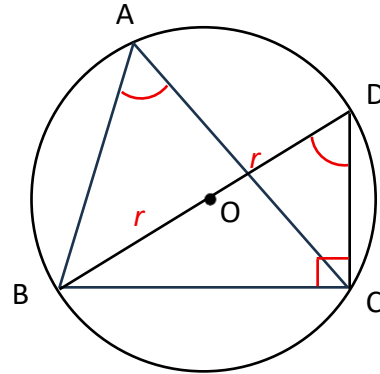
Prove that $r = \frac{a}{2\sin A}$.

Solution

Draw a diagram.

$$\begin{aligned}\angle BAC &= \angle BDC & (\angle s \text{ in same segment}) \\ \therefore \sin A &= \sin D\end{aligned}$$

$$\begin{aligned}\angle BCD &= 90^\circ & (\angle \text{ in semicircle}) \\ \sin D &= \frac{BC}{BD} \\ &= \frac{a}{2r} & (BD = 2r) \\ \therefore r &= \frac{a}{2\sin A}\end{aligned}$$



Question 2

In $\triangle ABC$, a line AD is drawn from the vertex A to the side BC , bisecting the interior angle A .

Prove that $\frac{BD}{CD} = \frac{c}{b}$

Solution

Draw a diagram.

Let $\angle BAC = 2\alpha$ and $\angle BDA = \theta$

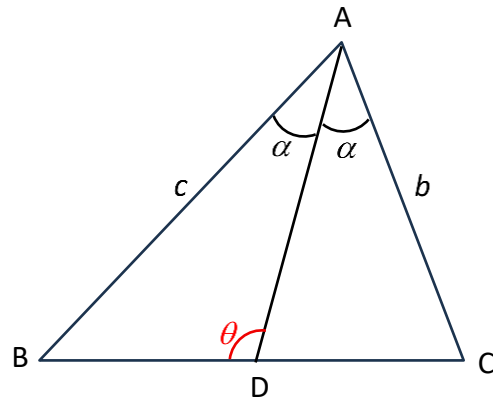
$$\text{Then } \frac{BD}{\sin \alpha} = \frac{c}{\sin \theta} \quad (\text{sine rule})$$

$$\sin \alpha = \frac{BD \sin \theta}{c} \quad (1)$$

In $\triangle ACD$,

$$\frac{CD}{\sin \alpha} = \frac{b}{\sin(180^\circ - \theta)} = \frac{b}{\sin \theta} \quad (\text{sine rule})$$

$$\therefore \sin \alpha = \frac{CD \sin \theta}{b} \quad (2)$$



From (1) and (2)

$$\frac{BD \sin \theta}{c} = \frac{CD \sin \theta}{b}$$

$$\therefore \frac{BD}{CD} = \frac{c}{b}$$

Question 3

Two observation platforms, A and B, are situated on a cliff face, with B a distance d vertically above A. From each platform, a fixed object C may be seen on the horizontal ground below. The angle of depression from B to C is β and that from A to C is α .

Prove that $AC = \frac{d \cos \beta}{\sin(\beta - \alpha)}$

Solution

Draw a diagram.

The angles of $\triangle ABC$ are:

$$\angle ABC = 90^\circ - \beta$$

$$\angle BAC = 90^\circ + \alpha$$

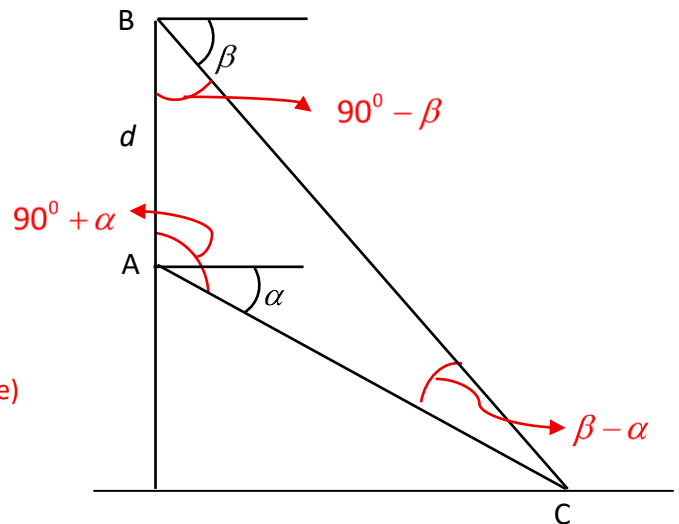
$$\angle ACB = \beta - \alpha$$

$$\frac{AC}{\sin(90^\circ - \beta)} = \frac{AB}{\sin(\beta - \alpha)}$$

$$\therefore AC = \frac{d \sin(90^\circ - \beta)}{\sin(\beta - \alpha)}$$

$$\therefore AC = \frac{d \cos \beta}{\sin(\beta - \alpha)}$$

(sine rule)



Question 4

A surveyor's vertical mast, TR, of unknown height h , stands on horizontal ground with its base at R. Two observation points, A and B, are laid out on the ground in a straight line extending from the base of the mast such that A, B and R are collinear and the distance AB is d . From point A the angle of elevation of the top of the mast, T, is α , while from point B, it is 2α .

Prove that the height of the mast may be expressed as $h = d \sin 2\alpha$.

Solution

Draw a diagram.

In $\triangle ABT$, $\angle TAB = \alpha$

The exterior $\angle TBR = 2\alpha$

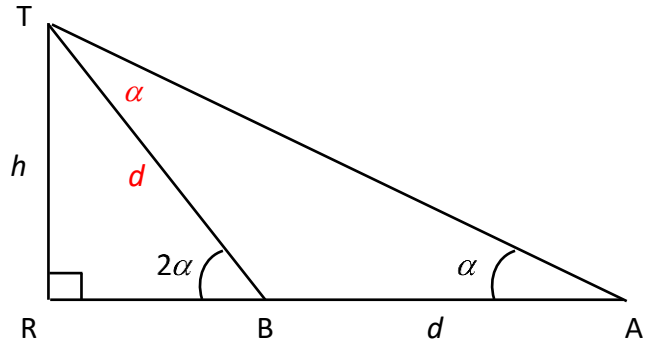
$$\therefore \angle BTA = \alpha$$

So, $BA = BT = d$

$$\sin TBR = \frac{TR}{TB}$$

$$\therefore \sin 2\alpha = \frac{h}{d}$$

$$\therefore h = d \sin 2\alpha$$



Question 5

A mountaineer starts at a point Q and measures the angle of elevation of the top of a mountain at P to be α . He then climbs a distance d metres at an incline of β ($\beta < \alpha$) to a point T and measures the angle of elevation of the mountain top again, which this time he finds to be θ . Show that the height of the mountain is

given by $PA = \frac{d \sin(\theta - \beta) \sin \alpha}{\sin(\theta - \alpha)}$.

Solution

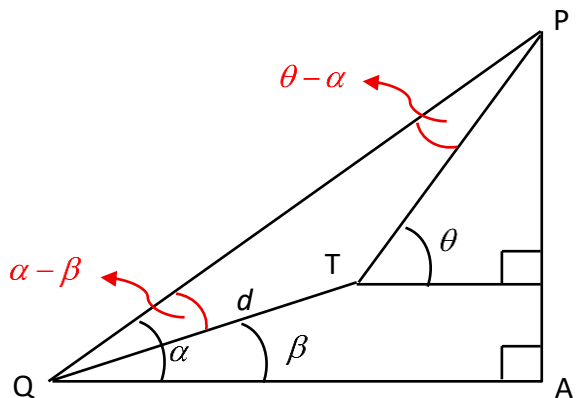
Draw a diagram.

$$\angle QPA = 90^\circ - \alpha$$

$$\angle TPA = 90^\circ - \theta$$

$$\therefore \angle QPT = \theta - \alpha$$

$$\therefore \angle PQT = \alpha - \beta$$



$$\begin{aligned} \therefore \angle QTP &= 180^\circ - (\theta - \alpha) - (\alpha - \beta) \\ &= 180^\circ - (\theta - \beta) \end{aligned}$$

$$\begin{aligned} \text{In } \triangle PQA, \frac{PA}{PQ} &= \sin \alpha \\ \therefore PA &= PQ \sin \alpha \end{aligned}$$

$$\text{In } \triangle PQT, \frac{PQ}{\sin[180^\circ - (\theta - \beta)]} = \frac{d}{\sin(\theta - \alpha)}$$

(sine rule)

$$\therefore \frac{PQ}{\sin(\theta - \beta)} = \frac{d}{\sin(\theta - \alpha)}$$

$$\therefore PQ = \frac{d \sin(\theta - \beta)}{\sin(\theta - \alpha)}$$

$$\therefore PA = \frac{d \sin(\theta - \beta) \sin \alpha}{\sin(\theta - \alpha)}$$