

TRIGONOMETRY 5: QUIZ SOLUTIONS

Question 1

Determine the general solution of the equation $\sin 2\theta - \cos \theta = 0$.

- A. $\theta = 90^\circ + k.360^\circ$, or $\theta = 30^\circ + k.360^\circ$, or $\theta = 150^\circ + k.360^\circ$, where $k \in \mathbb{Z}$.
- B. $\theta = 90^\circ + k.180^\circ$, or $\theta = 30^\circ + k.360^\circ$, or $\theta = 150^\circ + k.360^\circ$, where $k \in \mathbb{Z}$.
- C. $\theta = 30^\circ + k.180^\circ$, or $\theta = 60^\circ + k.360^\circ$, or $\theta = 120^\circ + k.360^\circ$, where $k \in \mathbb{Z}$.
- D. $\theta = 30^\circ + k.360^\circ$, or $\theta = 150^\circ + k.360^\circ$, where $k \in \mathbb{Z}$.

Solution

$$\begin{aligned}\sin 2\theta - \cos \theta &= 0 \\ \therefore 2\sin \theta \cos \theta - \cos \theta &= 0 \\ \therefore \cos \theta (2\sin \theta - 1) &= 0 \\ \therefore \cos \theta = 0 \text{ or } \sin \theta &= \frac{1}{2}\end{aligned}$$

Option 1

If $\cos \theta = 0$, then we know that $\theta = 90^\circ$ or 270°

$\therefore \theta = 90^\circ + k.180^\circ$

Option 2

Reference $\angle = 30^\circ$, +ve in quadrants 1 and 2

$\therefore \theta = 30^\circ + k.360^\circ$, or $\theta = 150^\circ + k.360^\circ$

and where $k \in \mathbb{Z}$.

So, the correct answer is B.

Question 2

Determine the general solution of the equation $\cos 2x + 3\sin x - 2 = 0$.

- A. $x = 30^\circ + k.360^\circ$, or $x = 150^\circ + k.360^\circ$, or $x = 90^\circ + k.360^\circ$, where $k \in \mathbb{Z}$.
- B. $x = 30^\circ + k.360^\circ$, or $x = 150^\circ + k.360^\circ$, where $k \in \mathbb{Z}$.
- C. $x = 60^\circ + k.360^\circ$, or $x = 120^\circ + k.360^\circ$, or $x = 90^\circ + k.360^\circ$, where $k \in \mathbb{Z}$.
- D. $x = 30^\circ + k.180^\circ$, or $x = 150^\circ + k.180^\circ$, or $x = 90^\circ + k.180^\circ$, where $k \in \mathbb{Z}$.

Solution

$$\begin{aligned}\cos 2x + 3\sin x - 2 &= 0 \\ \therefore (1 - 2\sin^2 x) + 3\sin x - 2 &= 0 \\ \therefore 2\sin^2 x - 3\sin x + 1 &= 0 \\ \therefore (2\sin x - 1)(\sin x - 1) &= 0\end{aligned}$$

$$\therefore \sin x = \frac{1}{2} \text{ or } \sin x = 1$$

Option 1

Reference $\angle = 30^\circ$, +ve in quadrants 1 and 2

$$\therefore x = 30^\circ + k.360^\circ \text{ or } x = 150^\circ + k.360^\circ \quad \text{or} \quad x = 90^\circ + k.360^\circ$$

Option 2

and where $k \in \mathbb{Z}$.

So, the correct answer is A.

Question 3

Determine the general solution of the equation $\sin 2\alpha + \sin \alpha = 0$.

A. $\alpha = 120^\circ + k.360^\circ$, or $\alpha = 240^\circ + k.360^\circ$, where $k \in \mathbb{Z}$.

B. $\alpha = k.180^\circ$, or $\alpha = 60^\circ + k.360^\circ$, or $\alpha = 120^\circ + k.360^\circ$, where $k \in \mathbb{Z}$.

C. $\alpha = k.360^\circ$, or $\alpha = 60^\circ + k.360^\circ$, or $\alpha = 300^\circ + k.360^\circ$, where $k \in \mathbb{Z}$.

D. $\alpha = k.180^\circ$, or $\alpha = 120^\circ + k.360^\circ$, or $\alpha = 240^\circ + k.360^\circ$, where $k \in \mathbb{Z}$.

Solution

$$\sin 2\alpha + \sin \alpha = 0$$

$$\therefore 2\sin \alpha \cos \alpha + \sin \alpha = 0$$

$$\therefore \sin \alpha (2\cos \alpha + 1) = 0$$

$$\therefore \sin \alpha = 0 \text{ or } \cos \alpha = -\frac{1}{2}$$

Option 1

If $\sin \alpha = 0$, then we know that $\alpha = 0^\circ, 180^\circ$, or 360°

$$\therefore \alpha = k.180^\circ$$

Option 2

Reference $\angle = 60^\circ$, -ve in quadrants 2 and 3

$$\alpha = 120^\circ + k.360^\circ, \text{ or } \alpha = 240^\circ + k.360^\circ$$

and where $k \in \mathbb{Z}$.

So, the correct answer is D.

Question 4

Find the general solution of the equation $4\cos^2 x - \cos 2x = 2$.

- A. $x = 45^\circ + k.180^\circ$, or $x = 135^\circ + k.180^\circ$, where $k \in \mathbb{Z}$.
- B. $x = 45^\circ + k.360^\circ$, or $x = 135^\circ + k.360^\circ$, where $k \in \mathbb{Z}$.
- C. $x = 45^\circ + k.90^\circ$, where $k \in \mathbb{Z}$.
- D. $x = 30^\circ + k.180^\circ$, or $x = 150^\circ + k.180^\circ$, where $k \in \mathbb{Z}$.

Solution

$$\begin{aligned} 4\cos^2 x - \cos 2x &= 2 \\ \therefore 4\cos^2 x - (2\cos^2 x - 1) &= 2 \\ \therefore 2\cos^2 x &= 1 \\ \therefore \cos^2 x &= \frac{1}{2} \\ \therefore \cos x &= \pm \frac{1}{\sqrt{2}} \end{aligned}$$

Reference $\angle = 45^\circ$, in all 4 quadrants

$$x = 45^\circ + k.90^\circ \quad [\text{or} \quad x = 45^\circ + k.180^\circ \text{ or } x = 135^\circ + k.180^\circ]$$

So, the correct answer is C.

Alternative solution

$$\begin{aligned} 4\cos^2 x - \cos 2x &= 2 \\ \therefore 4\cos^2 x - 2 - \cos 2x &= 0 \\ \therefore 2(2\cos^2 x - 1) - \cos 2x &= 0 \\ \therefore 2(\cos 2x) - \cos 2x &= 0 \\ \therefore \cos 2x &= 0 \\ \text{i.e.} \quad 2x &= 90^\circ \text{ or } 270^\circ \\ \therefore x &= 45^\circ \text{ or } 135^\circ \text{ as before.} \end{aligned}$$

Question 5

Which of the following is the general solution of the equation $\cos 2A + \cos A = 0$?

- A. $A = 60^\circ + k.120^\circ$, or $A = 180^\circ + k.360^\circ$, where $k \in \mathbb{Z}$.
- B. $A = 60^\circ + k.360^\circ$, or $A = 300^\circ + k.360^\circ$, or $A = 180^\circ + k.360^\circ$, where $k \in \mathbb{Z}$.
- C. $A = 60^\circ + k.180^\circ$, or $A = 180^\circ + k.180^\circ$, where $k \in \mathbb{Z}$.
- D. $A = 30^\circ + k.360^\circ$, or $A = 150^\circ + k.360^\circ$, or $A = 180^\circ + k.360^\circ$, where $k \in \mathbb{Z}$.

Solution

$$\cos 2A + \cos A = 0$$

$$\therefore 2\cos^2 A - 1 + \cos A = 0$$

$$\therefore 2\cos^2 A + \cos A - 1 = 0$$

$$\therefore (2\cos A - 1)(\cos A + 1) = 0$$

$$\therefore \cos A = \frac{1}{2} \text{ or } \cos A = -1$$

Option 1

Reference $\angle = 60^\circ$, +ve in quadrants 1 and 4

$$\therefore A = 60^\circ + k \cdot 360^\circ \text{ or } A = 300^\circ + k \cdot 360^\circ$$

or

Option 2

$$A = 180^\circ + k \cdot 360^\circ$$

and where $k \in \mathbb{Z}$.

So, the correct answer is B.