

TRIGONOMETRY 4: QUIZ SOLUTIONS

Question 1

Prove the identity $\frac{1 - \sin 2\theta}{\sin \theta - \cos \theta} = \sin \theta - \cos \theta$.

Solution

$$\begin{aligned} LHS &= \frac{1 - 2\sin \theta}{\sin \theta - \cos \theta} \\ &= \frac{(\sin^2 \theta + \cos^2 \theta) - 2\sin \theta \cos \theta}{\sin \theta - \cos \theta} && \sin^2 x + \cos^2 x = 1 \\ &= \frac{\sin^2 \theta - 2\sin \theta \cos \theta + \cos^2 \theta}{\sin \theta - \cos \theta} && a^2 - 2ab + b^2 = (a - b)^2 \\ &= \frac{(\sin \theta - \cos \theta)(\sin \theta - \cos \theta)}{(\sin \theta - \cos \theta)} \\ &= \sin \theta - \cos \theta \\ &= RHS \end{aligned}$$

Question 2

Prove that $\cos(A + 45^\circ) \cdot \cos(A - 45^\circ) = \frac{1}{2} \cos 2A$.

Solution

$$\begin{aligned} LHS &= \cos(A + 45^\circ) \cdot \cos(A - 45^\circ) \\ &= (\cos A \cos 45^\circ - \sin A \sin 45^\circ)(\cos A \cos 45^\circ + \sin A \sin 45^\circ) \\ &= \left(\frac{1}{\sqrt{2}} \cos A - \frac{1}{\sqrt{2}} \sin A \right) \left(\frac{1}{\sqrt{2}} \cos A + \frac{1}{\sqrt{2}} \sin A \right) \\ &= \left(\frac{1}{\sqrt{2}} \cos A \right)^2 - \left(\frac{1}{\sqrt{2}} \sin A \right)^2 && (a - b)(a + b) = a^2 - b^2 \\ &= \frac{1}{2} \cos^2 A - \frac{1}{2} \sin^2 A \\ &= \frac{1}{2} (\cos^2 A - \sin^2 A) \\ &= \frac{1}{2} \cos 2A \\ &= RHS \end{aligned}$$

Question 3

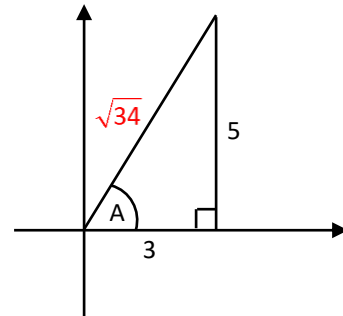
If $3 \tan A - 5 = 0$ and $0^\circ \leq A \leq 180^\circ$, show that $\tan 2A = -\frac{15}{8}$.

Solution

$$1. \quad 3 \tan A - 5 = 0$$

$$\therefore \tan A = \frac{5}{3}$$

$$\therefore \sin A = \frac{5}{\sqrt{34}} \text{ and } \cos A = \frac{3}{\sqrt{34}}$$



$$2. \quad \tan 2A = \frac{\sin 2A}{\cos 2A} = \frac{2 \sin A \cos A}{\cos^2 A - \sin^2 A}$$

$$3. \text{ From above } 2 \sin A \cos A = 2 \times \frac{5}{\sqrt{34}} \times \frac{3}{\sqrt{34}} = \frac{30}{34} = \frac{15}{17}$$

$$\cos^2 A - \sin^2 A = \left(\frac{3}{\sqrt{34}} \right)^2 - \left(\frac{5}{\sqrt{34}} \right)^2 = \frac{9}{34} - \frac{25}{34} = -\frac{16}{34} = -\frac{8}{17}$$

$$\therefore \tan 2A = \frac{15}{17} \times \left(-\frac{17}{8} \right) = -\frac{15}{8}$$

Question 4

Show that the exact value of $\cos 165^\circ$ is $-\frac{\sqrt{2+\sqrt{3}}}{2}$.

Solution

$$\cos 2A = 2 \cos^2 A - 1$$

Re-arranging this gives $2 \cos^2 A = 1 + \cos 2A$

$$\therefore \cos A = \pm \sqrt{\frac{1 + \cos 2A}{2}}$$

Now put $A = 165^\circ$, so that $2A = 330^\circ$

$$\text{This gives } \cos 165^\circ = -\sqrt{\frac{1 + \cos 330^\circ}{2}}$$

(165° is in the second quadrant, so $\cos 165^\circ$ is negative)

$$\begin{aligned} &= -\sqrt{\frac{1 + \cos 30^\circ}{2}} \\ &= -\sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} = -\frac{\sqrt{2+\sqrt{3}}}{2} \end{aligned}$$

Question 5

If $\sin 31^\circ = x$, show that $\cos 62^\circ = 1 - 2x^2$.

Solution

We know that $\cos(A + B) = \cos A \cos B - \sin A \sin B$.

Put $A = B$

Then $\cos 2A = \cos^2 A - \sin^2 A$ (or start here)

Now put $A = 31^\circ$

$$\therefore \cos 62^\circ = \cos^2 31^\circ - \sin^2 31^\circ$$

$$\begin{aligned} &= \left(\frac{\sqrt{1-x^2}}{1} \right)^2 - \left(\frac{x}{1} \right)^2 \\ &= 1 - x^2 - x^2 \\ &= 1 - 2x^2 \end{aligned}$$

