

TRIGONOMETRY 2: QUIZ SOLUTIONS

Question 1

Prove that $\sin(Q + R) - \sin(Q - R) = 2\sin Q \cos R$.

Solution

$$\begin{aligned} LHS &= \sin(Q + R) - \sin(Q - R) \\ &= (\sin Q \cos R + \cos Q \sin R) + (\sin Q \cos R - \cos Q \sin R) \\ &= \sin Q \cos R + \cos Q \sin R + \sin Q \cos R - \cos Q \sin R \\ &= 2\sin Q \cos R \\ &= RHS \end{aligned}$$

Question 2

Show that $\cos 105^\circ = -\frac{\sqrt{2}(\sqrt{3}-1)}{4}$.

Solution

$$\begin{aligned} \cos 105^\circ &= \cos(45^\circ + 60^\circ) \\ &= \cos 45^\circ \cos 60^\circ - \sin 45^\circ \sin 60^\circ \\ &= \frac{1}{\sqrt{2}} \cdot \frac{1}{2} - \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} \\ &= \frac{1 - \sqrt{3}}{2\sqrt{2}} \\ &= -\frac{\sqrt{2}(\sqrt{3}-1)}{4} \end{aligned}$$

Question 3

Prove that $\sin(120^\circ - \alpha) - \cos(330^\circ - \alpha) = 0$.

Solution

$$\begin{aligned} \sin(120^\circ - \alpha) &= \sin 120^\circ \cos \alpha - \cos 120^\circ \sin \alpha \\ &= \sin 60^\circ \cos \alpha - (-\cos 60^\circ) \sin \alpha \\ &= \sin 60^\circ \cos \alpha + \cos 60^\circ \sin \alpha \\ &= \frac{\sqrt{3}}{2} \cos \alpha + \frac{1}{2} \sin \alpha \\ \\ \cos(330^\circ - \alpha) &= \cos 330^\circ \cos \alpha + \sin 330^\circ \sin \alpha \\ &= \cos(-30^\circ) \cos \alpha - \sin(-30^\circ) \sin \alpha \\ &= \cos 30^\circ \cos \alpha + \sin 30^\circ \sin \alpha \\ &= \frac{\sqrt{3}}{2} \cos \alpha + \frac{1}{2} \sin \alpha \end{aligned}$$

$$\therefore \sin(120^\circ - \alpha) - \cos(330^\circ - \alpha) = \left(\frac{\sqrt{3}}{2} \cos \alpha + \frac{1}{2} \sin \alpha \right) - \left(\frac{\sqrt{3}}{2} \cos \alpha + \frac{1}{2} \sin \alpha \right) = 0$$

Question 4

Prove that $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$.

Solution

$$LHS = \frac{\sin(A+B)}{\cos(A+B)}$$

$$= \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}$$

$$= \frac{\frac{\sin A \cos B}{\cos A \cos B} + \frac{\cos A \sin B}{\sin A \sin B}}{\frac{\cos A \cos B}{\cos A \cos B} - \frac{\sin A \sin B}{\cos A \cos B}}$$

Divide each of the four terms by $\cos A \cos B$, then simplify.

$$= \frac{\tan A + \tan B}{1 - \tan A \tan B}$$
$$= RHS$$

Question 5

Prove that $\sin^2 X \cos^2 Y - \cos^2 X \sin^2 Y = \sin(X+Y) \cdot \sin(X-Y)$.

Solution

$$RHS = (\sin X \cos Y + \cos X \sin Y)(\sin X \cos Y - \cos X \sin Y)$$

This is in the form $(a+b)(a-b)$, with $a = \sin X \cos Y$ and $b = \cos X \sin Y$.

We know that $(a+b)(a-b) = a^2 - b^2$.

$$\therefore (\sin X \cos Y + \cos X \sin Y)(\sin X \cos Y - \cos X \sin Y)$$
$$= \sin^2 X \cos^2 Y - \cos^2 X \sin^2 Y$$
$$= LHS$$