

TRIGONOMETRY 1: QUIZ SOLUTIONS

Question 1

Which of the following is the simplified form of $\sin(\alpha - \beta) - \sin(\alpha + \beta)$?

- A. $2\cos\alpha\sin\beta$ B. $2\sin\alpha\cos\beta$
C. $-2\cos\alpha\sin\beta$ D. $-2\sin\alpha\cos\beta$

Solution

$$\begin{aligned} & \sin(\alpha - \beta) - \sin(\alpha + \beta) \\ &= (\sin\alpha\cos\beta - \cos\alpha\sin\beta) - (\sin\alpha\cos\beta + \cos\alpha\sin\beta) \\ &= \sin\alpha\cos\beta - \cos\alpha\sin\beta - \sin\alpha\cos\beta - \cos\alpha\sin\beta \\ &= -2\cos\alpha\sin\beta \end{aligned}$$

So, the correct answer is C.

Question 2

Simplify the expression $\sin(\theta - 60^\circ) + \cos(\theta + 30^\circ)$ (no calculator).

- A. $\sqrt{3} + 1$ B. 0 C. $\sqrt{3}\cos\theta$ D. $2\sin\theta$

Solution

$$\begin{aligned} & \sin(\theta - 60^\circ) + \cos(\theta + 30^\circ) \\ &= (\sin\theta\cos 60^\circ - \cos\theta\sin 60^\circ) + (\cos\theta\cos 30^\circ - \sin\theta\sin 30^\circ) \\ &= \sin\theta \cdot \frac{1}{2} - \cos\theta \frac{\sqrt{3}}{2} + \cos\theta \frac{\sqrt{3}}{2} - \sin\theta \cdot \frac{1}{2} \\ &= 0 \end{aligned}$$

So, the correct answer is B.

Question 3

Simplify the expression $\sin(x - 40^\circ)\sin(x + 20^\circ) + \cos(x - 40^\circ)\cos(x + 20^\circ)$.

- A. $\frac{1}{2}$ B. $-\frac{1}{2}$ C. $-\frac{\sqrt{3}}{2}$ D. $\frac{\sqrt{3}}{2}$

Solution

Put $A = x - 40^\circ$ and $B = x + 20^\circ$.

$$\begin{aligned} \text{Then } \sin(x - 40^\circ)\sin(x + 20^\circ) + \cos(x - 40^\circ)\cos(x + 20^\circ) \\ &= \sin A \sin B + \cos A \cos B \\ &= \cos A \cos B + \sin A \sin B \\ &= \cos(A - B) \\ &= \cos[(x - 40^\circ) - (x + 20^\circ)] \\ &= \cos(x - 40^\circ - x - 20^\circ) \\ &= \cos(-60^\circ) \\ &= \cos(60^\circ) \\ &= \frac{1}{2} \end{aligned}$$

So, the correct answer is A.

Question 4

Which of the following expressions is the simplified form of $\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta$?

- A. $\cos\theta$ B. $\sin\theta$
C. $\sin(\theta - 30^\circ)$ D. $\cos(\theta + 30^\circ)$

Solution

We know that $\cos(A + B) = \cos A \cos B - \sin A \sin B$

This means we can write

$$\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta = \cos A \cos\theta - \sin A \sin\theta$$

But if $\cos A = \frac{\sqrt{3}}{2}$ and $\sin A = \frac{1}{2}$, then $A = 30^\circ$

$$\begin{aligned} \therefore \frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta &= \cos 30^\circ \cos\theta - \sin 30^\circ \sin\theta \\ &= \cos(30^\circ + \theta) \end{aligned}$$

So, the correct answer is D.

Question 5

If $\cos A = -\frac{12}{13}$ with $0^\circ \leq A \leq 180^\circ$ and $\tan B = \frac{12}{5}$ with $180^\circ \leq B \leq 360^\circ$, what is the value of $\sin(A - B)$? (No calculator).

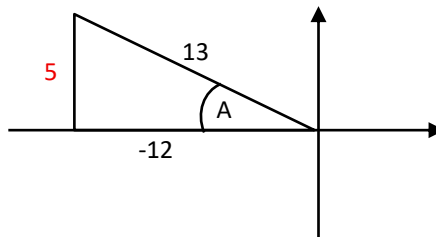
- A. $\frac{\sqrt{3}}{2}$ B. $\frac{1}{2}$ C. 0 D. -1

Solution

Given $\cos A = -\frac{12}{13}$ with $0^\circ \leq A \leq 180^\circ$.

This is a 5, 12, 13 triangle (Pythagoras) in the second quadrant.

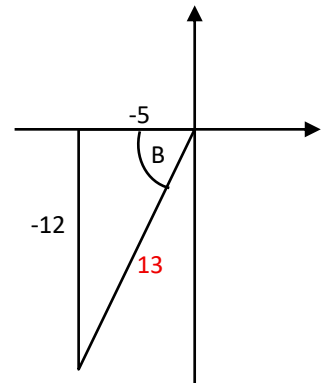
Then $\sin A = \frac{5}{13}$.



Given $\tan B = \frac{12}{5}$ with $180^\circ \leq B \leq 360^\circ$.

This is a 5, 12, 13 triangle (Pythagoras) in the third quadrant.

Then $\sin B = -\frac{12}{13}$ and $\cos B = -\frac{5}{13}$



$$\begin{aligned} \therefore \sin(A - B) &= \sin A \cos B - \cos A \sin B = \frac{5}{13} \times \left(-\frac{5}{13}\right) - \left(-\frac{12}{13}\right) \left(-\frac{12}{13}\right) \\ &= -\frac{25}{169} - \frac{144}{169} \\ &= -\frac{169}{169} = -1 \end{aligned}$$

So, the correct answer is D.