

ANALYTICAL GEOMETRY 4: QUIZ SOLUTIONS

Question 1

A circle has its centre at the point (9; 2). What is the equation of the tangent to the circle at the point (11; 5)?

- A. $y = -\frac{2}{3}x + \frac{22}{3}$ B. $y = -\frac{2}{3}x + \frac{37}{3}$
 C. $y = \frac{3}{2}x + \frac{33}{2}$ D. $y = \frac{3}{2}x + \frac{47}{2}$

Solution

Draw a diagram showing the circle and the two points.

Now add the **radius** and the **tangent**.

Gradient of radius is $\frac{5-2}{11-9} = \frac{3}{2}$

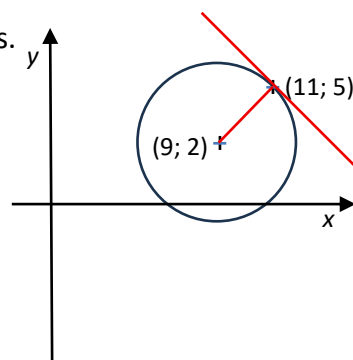
Radius and tangent are perpendicular.

So, gradient of tangent is $-\frac{2}{3}$.

Equation of tangent is $(y-5) = -\frac{2}{3}(x-11)$

$$y = -\frac{2}{3}x + \frac{22}{3} + 5$$

$$\therefore y = -\frac{2}{3}x + \frac{37}{3}$$



So, the correct answer is B.

Question 2

What is the equation of the tangent to the circle $x^2 + y^2 - 4x - 6y - 3 = 0$ at the point (-2; 3)?

- A. $y = -1$ B. $y = 3$ C. $x = -2$ D. $x = 2$

Solution

$$x^2 + y^2 - 4x - 6y - 3 = 0$$

Rearrange terms $x^2 - 4x + y^2 - 6y - 3 = 0$

Complete the squares $(x-2)^2 - 4 + (y-3)^2 - 9 - 3 = 0$

Simplify/tidy up $(x-2)^2 + (y-3)^2 = 16$

This represents a circle with centre (2; 3) and radius 4.

So, the centre is at (2; 3) and the point of tangency is (-2; 3).

$$(-2; 3) \text{ ————— } (2; 3)$$

Notice anything special? Yes, y-coordinates are the same.

This means that the radius is horizontal.

And the tangent at (-2; 3) must be vertical.

So, the equation of the tangent at (-2; 3) is $x = -2$ (-2; 3) ————— (2; 3)

So, the correct answer is C.

Look carefully at the answer options. Did you notice they are lines parallel to either the x-axis or the y-axis?

Question 3

What is the equation of the tangent to the circle $x^2 + y^2 + 6x - 2y - 7 = 0$ at the point (-2; 5)?

A. $y = -\frac{1}{4}x + \frac{9}{2}$

B. $y = 4x - \frac{9}{2}$

C. $y = -\frac{4}{5}x + 5$

D. $y = \frac{5}{4}x - 5$

Solution

$$x^2 + y^2 + 6x - 2y - 7 = 0$$

Rearrange terms $x^2 + 6x + y^2 - 2y - 7 = 0$

Complete the squares $(x+3)^2 - 9 + (y-1)^2 - 1 - 7 = 0$

Simplify/tidy up $(x+3)^2 + (y-1)^2 = 9 + 1 + 7 = 17$

This represents a circle with centre (-3; 1) and radius $\sqrt{17}$.

Gradient of radius $= \frac{5-1}{-2-(-3)} = 4$

So, gradient of tangent $= -\frac{1}{4}$

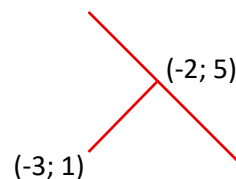
Equation of tangent is $y - 5 = -\frac{1}{4}(x - (-2))$

$$\therefore y - 5 = -\frac{1}{4}(x + 2)$$

$$\therefore y = -\frac{1}{4}x - \frac{1}{2} + 5$$

$$\therefore y = -\frac{1}{4}x + \frac{9}{2}$$

So, the correct answer is A.



Question 4

AB is the diameter of a circle, with A the point (0; 6) and B the point (6; -2). What is the equation of the tangent to the circle at (0; -2)?

A. $y = \frac{3}{4}x + 6$

B. $y = \frac{3}{4}x - \frac{13}{2}$

C. $y = -\frac{3}{4}x + \frac{13}{2}$

D. $y = -\frac{3}{4}x - 2$

Solution

We need to know the coordinates of the centre.

Using the formula, the coordinates of C, the mid-point of AB, are:

$$x_c = \frac{0+6}{2} = 3 \text{ and } y_c = \frac{6+(-2)}{2} = 2$$

Draw a diagram showing the centre, the tangent point, the radius and the tangent.

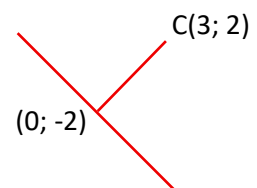
$$\text{Gradient of radius} = \frac{2 - (-2)}{3 - 0} = \frac{4}{3}$$

$$\text{So, gradient of tangent} = -\frac{3}{4}$$

$$\text{Equation of tangent is } y - (-2) = -\frac{3}{4}(x - 0)$$

$$\therefore y = -\frac{3}{4}x - 2$$

So, the correct answer is D.

**Question 5**

The equation of a circle is $x^2 + y^2 - 10x - 3 = 0$. Does this circle touch the y-axis, cut the y-axis, or neither touch nor cut the y-axis?

A. Touch

B. Cut

C. Neither

Solution

$$x^2 + y^2 - 10x - 3 = 0$$

$$\text{Rearrange terms} \quad x^2 - 10x + y^2 - 3 = 0$$

$$\text{Complete the square} \quad (x - 5)^2 - 25 + y^2 - 3 = 0$$

$$\text{Simplify/tidy up} \quad (x - 5)^2 + y^2 = 28$$

This represents a circle with centre (5; 0) and $r^2 = 28$

Distance from (0; 0) to (5; 0) is 5.

But $5 < \sqrt{28}$, so circle will cut y-axis in two points.

So, the correct answer is B.

