AnalyticalGeometry 1QuizSolutions.doc

ANALYTICAL GEOMETRY 3: QUIZ SOLUTIONS

Question 1

Which of the following is the equation of a circle with centre (2; 1) and radius 3?

A. $x^2+y^2+4x+2y+4=0$ B. $x^2+y^2+2x+y+3=0$

C. $x^2+y^2-2x-y-3=0$ D. $x^2+y^2-4x-2y-4=0$

Solution

The standard equation of a circle with its centre at $(a; b)$ and radius r is

$$(x-a)^2+(y-b)^2=r^2.$$

For a circle, centre (2; 1), radius 3, we have

$$(x-2)^2+(y-1)^2=3^2$$
$$\therefore x^2-4x+4+y^2-2y+1=9$$
$$\therefore x^2+y^2-4x-2y-4=0$$

So, the correct answer is D

Question 2

What is the centre of the circle $x^2+y^2+2x-6y-3=0$?

A. $(-1; 3)$ B. $(1; -3)$ C. $(2; -6)$ D. $(-2; 6)$

Solution

$$x^2+y^2+2x-6y-3=0$$

Rearrange terms $x^2+2x+y^2-6y-3=0$

Complete the squares $(x+1)^2-1+(y-3)^2-9-3=0$

Simplify/tidy up $(x+1)^2+(y-3)^2=13$

This represents a circle with centre $(-1; 3)$ and radius $\sqrt{13}$

So, the correct answer is A.

Question 3

What is the radius of the circle $x^2+y^2+3x-5y+1=0$?

A. $\frac{15}{2}$ B. $\sqrt{\frac{15}{2}}$ C. $\frac{19}{2}$ D. $\sqrt{\frac{19}{2}}$

Solution

$$x^2+y^2+3x-5y+1=0$$

Rearrange terms $x^2+3x+y^2-5y+1=0$

Complete the squares $\left(x+\frac{3}{2}\right)^2-\frac{9}{4}+\left(y-\frac{5}{2}\right)^2-\frac{25}{4}+1=0$

Simplify/tidy up $\left(x+\frac{3}{2}\right)^2+\left(y-\frac{5}{2}\right)^2=\frac{9}{4}+\frac{25}{4}-1=\frac{30}{4}=\frac{15}{2}$

This represents a circle with centre $\left(-\frac{3}{2};\frac{5}{2}\right)$ and radius $\sqrt{\frac{15}{2}}$

So, the correct answer is B.

Question 4

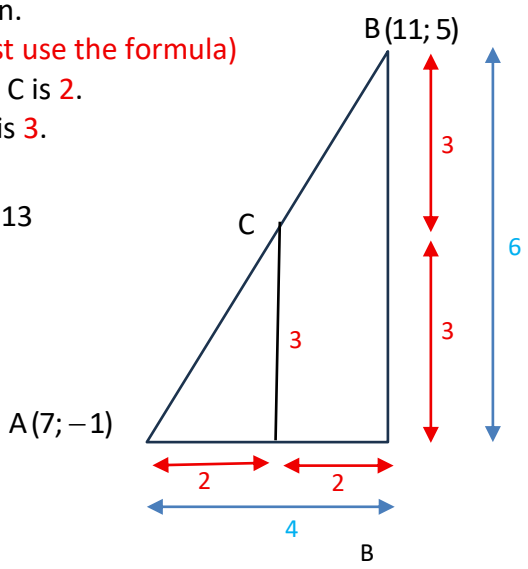
The points $A(7;-1)$ and $B\ (11;5)$ are the ends of the diameter of a circle. What is the equation of the circle?

A. $x^2+y^2+4x+6y+10=0$ B. $x^2+y^2-2x-3y-5=0$

C. $x^2+y^2-18x-4y+72=0$ D. $x^2+y^2-9x-2y+2=0$

Solution

We need to know the coordinates of the centre (let's call it C) and the radius.
There is more than one way to tackle this question.
A simple intuitive way is to draw a diagram (or just use the formula)
Horizontally, the distance from A to B is 4, so A to C is 2.
Vertically, the distance from A to B is 6, so A to C is 3.
This means the coordinates of C are $(9;2)$



And using a right-angled triangle, $2^2+3^2=4+9=13$
So now we know that $r^2=13$.

$$\therefore (x-9)^2+(y-2)^2=13$$
$$\therefore x-18x+81+y^2-4y+4=13$$
$$\therefore x^2+y^2-18x-4y+72=0$$

So. the correct answer is C.

Question 5

The equation of a circle is $x^2 + y^2 - 6x - 4y - 12 = 0$. Does the point (5; 6) inside, outside, or on the circle?

A. Inside the circle B. Outside the circle C. On the circle

Solution

$$x^2 + y^2 - 6x - 4y - 12 = 0$$

Rearrange terms $x^2 - 6x + y^2 - 4y - 12 = 0$

Complete the squares $(x-3)^2 - 9 + (y-2)^2 - 4 - 12 = 0$

Simplify/tidy up $(x-3)^2 + (y-2)^2 = 25$

This represents a circle with centre $(3;2)$ and $r^2 = 25$

Distance *d* from (3; 2) to (5; -6) is given by

$d^2 = (5-3)^2 + (6-2)^2 = 2^2 + 4^2 = 4 + 16 = 20$ Easier not to bother with square roots

Since 20 < 25, point (5; 6) must be inside the circle.

So, the correct answer is A.