

ANALYTICAL GEOMETRY 3: QUIZ SOLUTIONS

Question 1

Which of the following is the equation of a circle with centre (2; 1) and radius 3?

A. $x^2 + y^2 + 4x + 2y + 4 = 0$

B. $x^2 + y^2 + 2x + y + 3 = 0$

C. $x^2 + y^2 - 2x - y - 3 = 0$

D. $x^2 + y^2 - 4x - 2y - 4 = 0$

Solution

The standard equation of a circle with its centre at $(a; b)$ and radius r is

$$(x - a)^2 + (y - b)^2 = r^2.$$

For a circle, centre (2; 1), radius 3, we have

$$(x - 2)^2 + (y - 1)^2 = 3^2$$

$$\therefore x^2 - 4x + 4 + y^2 - 2y + 1 = 9$$

$$\therefore x^2 + y^2 - 4x - 2y - 4 = 0$$

So, the correct answer is D

Question 2

What is the centre of the circle $x^2 + y^2 + 2x - 6y - 3 = 0$?

A. $(-1; 3)$

B. $(1; -3)$

C. $(2; -6)$

D. $(-2; 6)$

Solution

$$x^2 + y^2 + 2x - 6y - 3 = 0$$

Rearrange terms $x^2 + 2x + y^2 - 6y - 3 = 0$

Complete the squares $(x + 1)^2 - 1 + (y - 3)^2 - 9 - 3 = 0$

Simplify/tidy up $(x + 1)^2 + (y - 3)^2 = 13$

This represents a circle with centre $(-1; 3)$ and radius $\sqrt{13}$

So, the correct answer is A.

Question 3

What is the radius of the circle $x^2 + y^2 + 3x - 5y + 1 = 0$?

- A. $\frac{15}{2}$ B. $\sqrt{\frac{15}{2}}$ C. $\frac{19}{2}$ D. $\sqrt{\frac{19}{2}}$

Solution

$$x^2 + y^2 + 3x - 5y + 1 = 0$$

Rearrange terms $x^2 + 3x + y^2 - 5y + 1 = 0$

Complete the squares $\left(x + \frac{3}{2}\right)^2 - \frac{9}{4} + \left(y - \frac{5}{2}\right)^2 - \frac{25}{4} + 1 = 0$

Simplify/tidy up $\left(x + \frac{3}{2}\right)^2 + \left(y - \frac{5}{2}\right)^2 = \frac{9}{4} + \frac{25}{4} - 1 = \frac{30}{4} = \frac{15}{2}$

This represents a circle with centre $\left(-\frac{3}{2}; \frac{5}{2}\right)$ and radius $\sqrt{\frac{15}{2}}$

So, the correct answer is B.

Question 4

The points A(7; -1) and B(11; 5) are the ends of the diameter of a circle. What is the equation of the circle?

- A. $x^2 + y^2 + 4x + 6y + 10 = 0$ B. $x^2 + y^2 - 2x - 3y - 5 = 0$
C. $x^2 + y^2 - 18x - 4y + 72 = 0$ D. $x^2 + y^2 - 9x - 2y + 2 = 0$

Solution

We need to know the coordinates of the centre (let's call it C) and the radius.

There is more than one way to tackle this question.

A simple intuitive way is to draw a diagram (or just use the formula)

Horizontally, the distance from A to B is 4, so A to C is 2.

Vertically, the distance from A to B is 6, so A to C is 3.

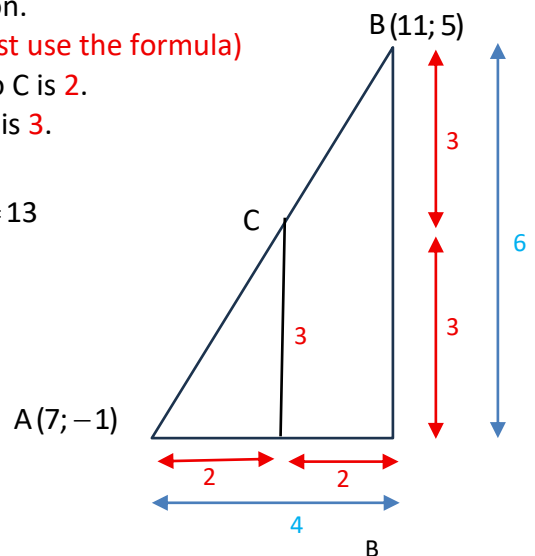
This means the coordinates of C are (9; 2)

And using a right-angled triangle, $2^2 + 3^2 = 4 + 9 = 13$

So now we know that $r^2 = 13$.

$$\begin{aligned}\therefore (x-9)^2 + (y-2)^2 &= 13 \\ \therefore x^2 - 18x + 81 + y^2 - 4y + 4 &= 13 \\ \therefore x^2 + y^2 - 18x - 4y + 72 &= 0\end{aligned}$$

So, the correct answer is C.



Question 5

The equation of a circle is $x^2 + y^2 - 6x - 4y - 12 = 0$. Does the point (5; 6) inside, outside, or on the circle?

A. Inside the circle

B. Outside the circle

C. On the circle

Solution

$$x^2 + y^2 - 6x - 4y - 12 = 0$$

Rearrange terms

$$x^2 - 6x + y^2 - 4y - 12 = 0$$

Complete the squares $(x - 3)^2 - 9 + (y - 2)^2 - 4 - 12 = 0$

Simplify/tidy up

$$(x - 3)^2 + (y - 2)^2 = 25$$

This represents a circle with centre (3; 2) and $r^2 = 25$

Distance d from (3; 2) to (5; -6) is given by

$$d^2 = (5 - 3)^2 + (-6 - 2)^2 = 2^2 + 4^2 = 4 + 16 = 20 \quad \text{Easier not to bother with square roots}$$

Since $20 < 25$, point (5; 6) must be inside the circle.

So, the correct answer is A.