

ANALYTICAL GEOMETRY 2: QUIZ SOLUTIONS

Question 1

The equation of a circle is $x^2 + y^2 = 25$. What is the equation of the tangent at the point $(-3; 4)$?

A. $y = \frac{4}{3}x + \frac{25}{3}$

B. $y = \frac{3}{4}x + \frac{25}{4}$

C. $y = -\frac{4}{3}x - \frac{25}{3}$

D. $y = -\frac{3}{4}x - \frac{25}{4}$

Solution

The standard equation of a circle with its centre at the origin is $x^2 + y^2 = r^2$.

For A, $x^2 + y^2 = 13$ is the standard equation of a circle, centre the origin.

For B, $x^2 - 2x + y^2 = 4$ is not the standard form of a circle, centre the origin.

For C, $x^2 + y^2 + 4y = 4$ is not the standard form of a circle, centre the origin.

For D, $x^2 - y^2 = 16$ is not the equation of a circle.

So, the correct answer is A.

Question 2

The equation of a circle is $x^2 + y^2 = 13$. If the straight-line AB is a diameter with A the point $(-2; 3)$, what are the coordinates of B??

A. $(-2; -3)$

B. $(-3; -2)$

C. $(2; -3)$

D. $(2; 3)$

Solution

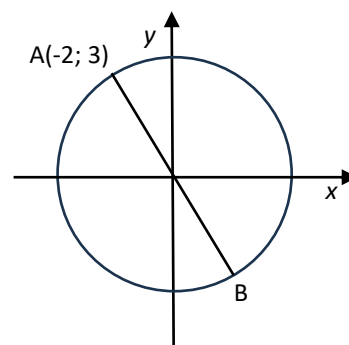
The simplest way here is to use symmetry.

Using symmetry about the y-axis means that -2 becomes +2.

Using symmetry about the x-axis means that +3 becomes -3.

This gives B as the point $(2; -3)$.

So, the correct answer is C.



x

Question 3

The equation of a circle is $x^2 + y^2 = 169$. Does the point $(10; -7)$ lie inside, outside, or on the circle?

- A. Inside the circle B. Outside the circle C. On the circle

Solution

$x^2 + y^2 = 169$ is the equation of a circle, centre the origin, radius 13.

In other words, the distance from the origin to any point on the circumference of the circle is 13.

Distance from the origin to the point $(10; -7)$ is

$$(10 - 0)^2 + (-7 - 0)^2 = 100 + 49 = 149$$

But $149 < 169$, or $\sqrt{149} < \sqrt{169}$ if you prefer.

This means that the point $(10; -7)$ must lie inside the circle.

So, the correct answer is A.

Question 4

Is the straight line $y = x - 1$ a tangent or a secant to the circle $x^2 + y^2 = 25$, or neither?

- A. Tangent B. Secant C. Neither

Solution

There is more than one way to tackle this question.

A simple intuitive way is to draw a diagram.

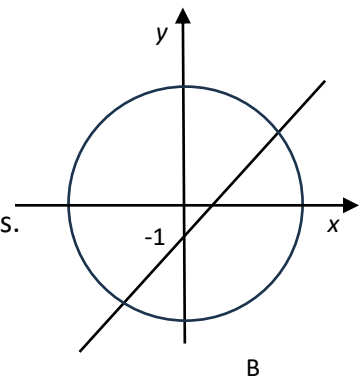
The circle has its centre at the origin and radius 5

The straight line has a y -intercept at -1 and its gradient is 1.

The straight line clearly cuts the circle in two distinct places.

This makes it a secant.

So the correct answer is B.



A longer, more formal, analytical method is to find the points of intersection, if any, between the circle and the straight line.

$$x^2 + y^2 = 25 \text{ and } y = x - 1$$

$$\therefore x^2 + (x - 1)^2 = 25$$

$$\therefore x^2 + x^2 - 2x + 1 = 25$$

$$\therefore 2x^2 - 2x - 24 = 0$$

$$\therefore x^2 - x - 12 = 0$$

$$\therefore (x + 3)(x - 4) = 0$$

$$\therefore x = -3 \text{ or } x = 4$$

This shows that there are two points of intersection, so the line is a secant.

Question 5

What is the equation of a circle with centre $(0; 0)$ that also passes through the point $(-4; 2)$?

A. $x^2 + y^2 = 6$

B. $x^2 + y^2 = 10$

C. $x^2 + y^2 = 16$

D. $x^2 + y^2 = 20$

Solution

The standard equation of a circle with its centre at the origin is $x^2 + y^2 = r^2$.

The distance from $(0; 0)$ to the point $(-4; 2)$ is the radius.

$$(-4 - 0)^2 + (2 - 0)^2 = 16 + 4 = 20$$

Equation of circle is $x^2 + y^2 = 20$

So, the correct answer is D.