TRIGONOMETRY EXAMPLE 1

Which of the following equations have no solutions?

(i) $2\cos x = 3$
(ii) $\sec x = \sin x$
(iii) $\sin x = \operatorname{cosec} x$
(iv) $\sin x - \cos x = 0$

(A) (iii) and (iv) only
(B) (i) and (ii) only
(C) (i), (ii) and (iv) only
(D) (ii), (iii)and (iv) only)

SOLUTION

(i) $2\cos x = 3$

$$\therefore \cos x = \frac{3}{2}$$

But we know that $-1 \le \cos x \le 1$
This means that (i) has no solutions.

(ii) $\sec x = \sin x$

$$\text{But } \sec x = \frac{1}{\cos x}$$
$$\therefore \sin x \cos x = 1$$
$$\text{or } 2\sin x \cos x = 2$$
$$\text{so } \sin 2x = 2$$

But we know that $-1 \le \sin 2x \le 1$
This means that (ii) has no solutions.

(iii) $\sin x = \operatorname{cosec} x$

$$\text{But } \operatorname{cosec} x = \frac{1}{\sin x}$$
$$\therefore \sin^2 x = 1$$
$$\therefore \sin x = \pm 1$$
$$\therefore x = 90^0 \text{ for example.}$$

This means that (iii) has at least one solution.

(iv) $\sin x - \cos x = 0$

$$\therefore \frac{\sin x}{\cos x} - \frac{\cos x}{\cos x} = 0$$

$$\therefore \tan x = 1$$

$$\therefore x = 45^0 \text{ for example.}$$

This means that (iv) has at least one solution.

In summary, (i) has no solutions
(ii) has no solutions
(iii) has at least one solution
(iv) has at least one solution

So, the correct answer is B.