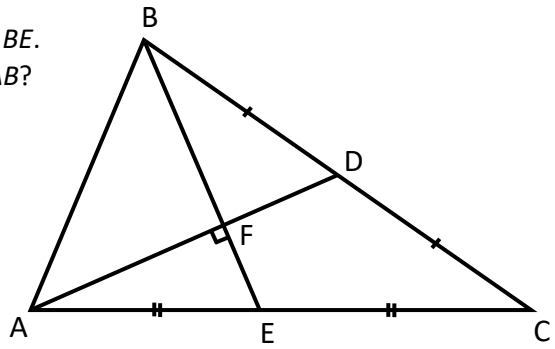
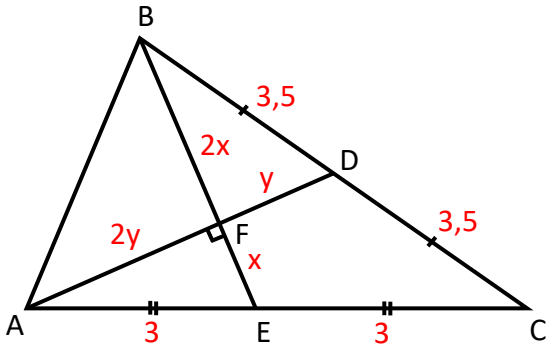
GEOMETRY EXAMPLE 3

In ΔABC, median *AD* is perpendicular to median *BE*.
If *AC* = 6 cm and *BC* = 7 cm, what is the length of *AB*?



(A) 4 cm (B) $\sqrt{17}$ cm (C) $2\sqrt{5}$ cm (D) 4,5 cm

SOLUTION



Let $EF = x$ and $DF = y$
$\therefore BF = 2x$ and $AF = 2y$
(medians trisect each other)

In ΔAFE, $x^2 + (2y)^2 = 3^2$
$\therefore x^2 + 4y^2 = 9$

In ΔBFD, $(2x)^2 + y^2 = 3{,}5^2 = \left(\frac{7}{2}\right)^2$
$\therefore 4x^2 + y^2 = \frac{49}{4}$

Adding these two equations gives

$$5x^2 + 5y^2 = \frac{85}{4}$$

Now multiply by 4 and divide by 5 to obtain

$$4x^2 + 4y^2 = 17$$

But in ΔABF, $AB^2 = 4x^2 + 4y^2$
$\therefore AB = \sqrt{17}$

So, the correct answer is B.