COMPOSITE TEST SOLUTIONS 37-40

Question 37

A playing card is drawn at random from a standard pack of 52 and at the same time a regular six-sided die is thrown. What is the probability that one of these actions, but not both, will result in a four?

A. $\frac{1}{78}$ B. $\frac{7}{78}$ C. $\frac{17}{78}$ D. $\frac{19}{78}$

Solution

There are two possibilities.

Card is a 4 and die is *not* a 4

Card is *not* a 4 and die is a 4

P(Card is a 4) $= \frac{4}{52} = \frac{1}{13}$

P(Card is *not* a 4) $= \frac{4}{52} = \frac{1}{13}$

P(Die is a 4) $= \frac{1}{6}$

P(Die is *not* a 4) $= \frac{5}{6}$

P(Card is a 4 *and* Die is not a 4) $= \frac{1}{13} \times \frac{5}{6} = \frac{5}{78}$

P(Card is not a 4 *and* Die is a 4) $= \frac{12}{13} \times \frac{1}{6} = \frac{12}{78}$

P(One of these actions) $= \frac{5}{78} + \frac{12}{78} = \frac{17}{78}$

So, the correct solution is C.

Question 38

If $\cos(\alpha+\beta)=0$, then what is $\sin(\alpha-\beta)$?

A. $\cos\beta$ B. $\cos 2\beta$ C. $\sin\alpha$ D. $\sin 2\alpha$

Solution

$$\cos(\alpha+\beta)=0$$

$$\therefore \alpha+\beta=90^0$$

$$\therefore \alpha=90^0-\beta$$

$$\therefore \sin(\alpha-\beta)=\sin(90^0-\beta-\beta)$$

$$=\sin(90^0-2\beta)$$

$$=\cos 2\beta$$

So, the correct answer is B.

Question 39

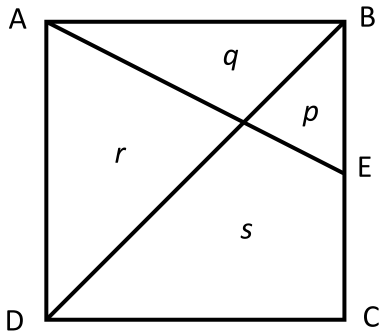
In the diagram, E is the mid-point of BC and ABCD is a square.

The areas of the four regions are *p*, *q*, *r* and s as shown.

What is the ratio *p* : *q* : *r* : *s*?

A. 3 : 5: 12: 14 B. 2 : 3 : 8 : 9

C. 1 : 2 : 6 : 7 D. 1 : 2 : 4 : 5

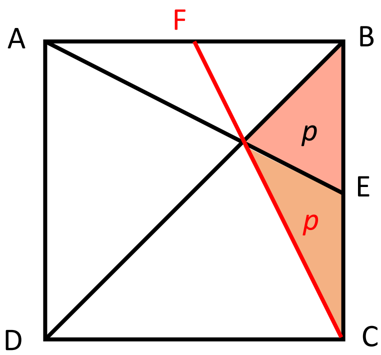


Solution

Draw CF symmetric to AE about BD.

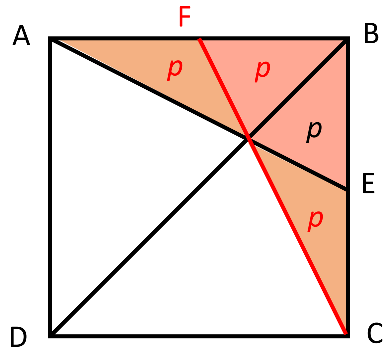
These two triangles have equal areas.

(equal bases, same height)



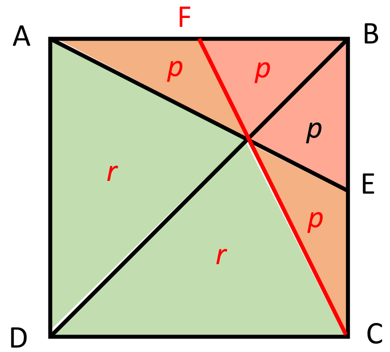
By symmetry, these two triangles also have equal areas.

(equal bases, same height)

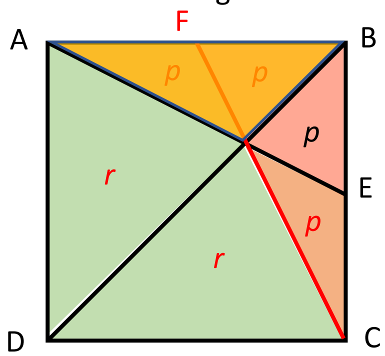


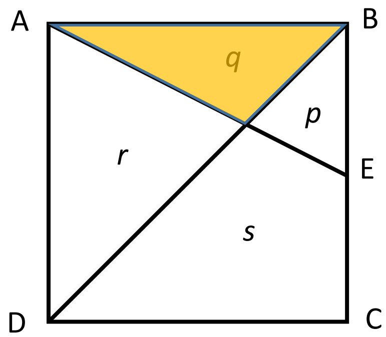
By symmetry, these two triangles have equal areas.

(equal bases, same height)



Now compare the shaded diagram with the original one.

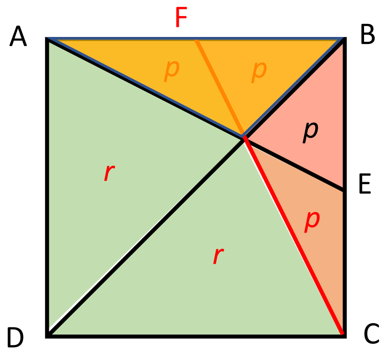


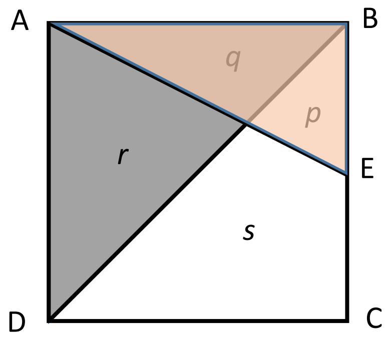


$$\therefore q = 2p$$

Area $\Delta ABD = 2\times$ Area ΔABE

(same base AB, AD = 2 x BE)

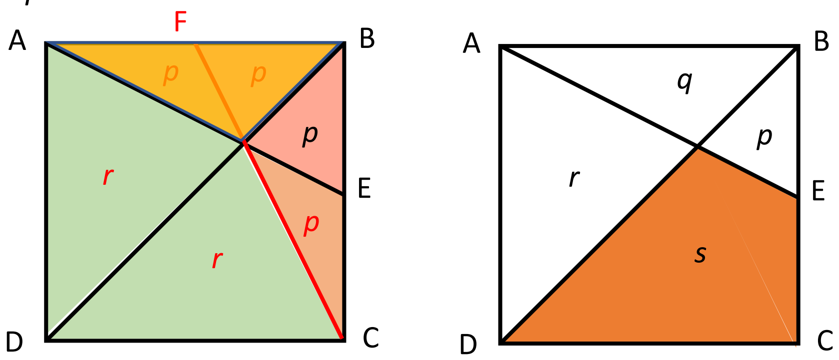




$$\therefore r + q = 2(p + q)$$

$$\therefore r = 2p + q \quad = 4p$$

Finally, $s = r + p$



$$\therefore s = 4p + p$$

$$= 5p$$

This means that the four areas are *p*, 2*p*, 4*p* and 5*p*.

The ratio is $1:2:4:5$

So, the correct answer is D.

Question 40

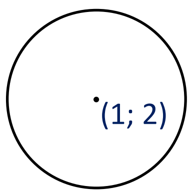
The line $x + y + k = 0$ touches the circle $(x-1)^2 + (y-2)^2 = 2$.

What are the possible values of *k*?

A. (-1; -5) B. (-1; 5) C. (1; -5) D. (1; 5)

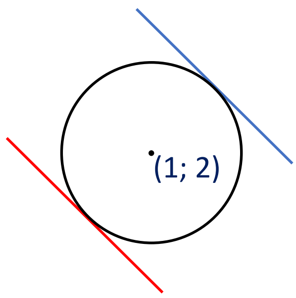
Solution

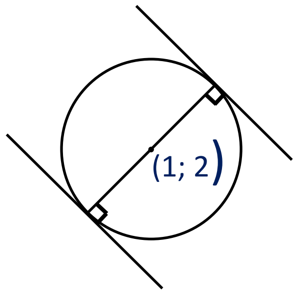
Circle $(x-1)^2 + (y-2)^2 = 2$ has centre (1; 2) and radius $\sqrt{2}$.



If $x + y + k = 0$ then $y = -x - k$.

The gradient of this line is -1. (two possibilities)





Diameter is perpendicular to the tangent.

$\therefore$ Diameter has a gradient of 1.

Equation of diameter is $y-2=1(x-1)$

$$\therefore y=x+1$$

Intersection of diameter and circle is given by

$$(x-1)^2+(x+1-2)^2=2$$

$$\therefore (x-1)^2=1$$

$$\therefore x=0 \text{ or } 2$$

$$\text{and } y=1 \text{ or } 3$$

But $k=-x-y$

$$\therefore k=0-1=-1$$

$$\text{or } k=-2-3=-5$$

So, the correct answer is A.