ALGEBRA TEST SOLUTIONS

Question 1

If the graph of $y = kx^2 + 3x - 2$ does not cut or touch the *x*-axis, what are the values of *k*?

A. $k > -\frac{9}{8}$ B. $k \geq -\frac{9}{8}$ C. $k \leq -\frac{9}{8}$ D. $k < -\frac{9}{8}$

Solution

If the graph of $y = kx^2 + 3x - 2$ does not cut or touch the *x*-axis, then there are no roots.

This means that the discriminant is negative.

$$\therefore b^2 - 4ac < 0$$

$$\therefore (3)^2 - 4(k)(-2) < 0$$

$$\therefore 9 + 8k < 0$$

$$\therefore k < -\frac{9}{8}$$

So, the correct answer is D.

Question 2

How many values of *x* satisfy the equation $\frac{2x^2 - 10x}{x^2 - 5x} = x - 3$?

A. 0 B. 1 C. 3 D. 4

Solution

The equation is meaningful only when the denominator is not zero.

$$\therefore x^2 - 5x \neq 0$$

$$\therefore x(x-5) \neq 0$$

$$\therefore x \neq 0 \text{ or } x \neq 5$$

Now we can simplify the left-hand side: $\frac{2x^2 - 10x}{x^2 - 5x} = \frac{2(x^2 - 5x)}{(x^2 - 5x)} = \frac{2\cancel{(x^2 - 5x)}^{1}}{\cancel{(x^2 - 5x)}^{1}} = 2$

$$\therefore 2 = x - 3$$

$$\therefore x = 5$$

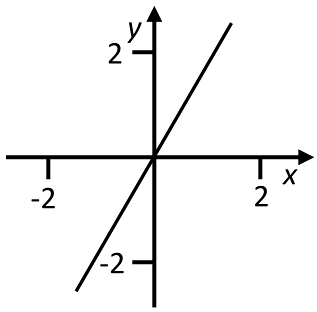
But we saw that $x \neq 5$, which means that there are no solutions.

So, the correct answer is A.

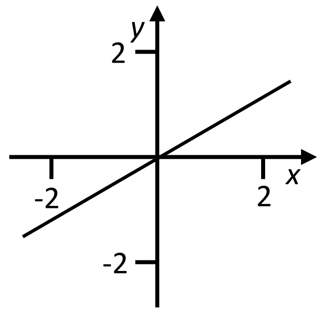
Question 3

Which graph represents the equation $2x + y = 0$?

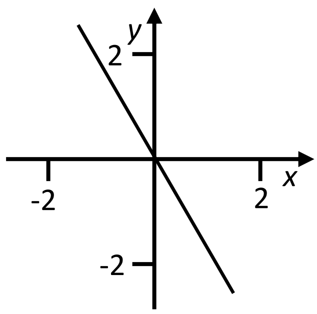
A.



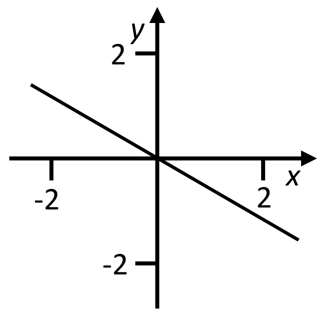
B.



C.



D.



Solution

If $2x + y = 0$, then $y = -2x$.

This is an equation that passes through the origin and has a gradient of -2.

So, the correct answer is C.

Question 4

The roots of the equation $|x + a| = b$ are 2 and -1. What is the value of b?

A. 10   B. $\frac{3}{2}$ C. $\frac{1}{2}$ D. -3

Solution

There's more than one method. Let's try squaring both sides and simplifying.

$$(|x + a|)^2 = b^2$$

$$\therefore x^2 + 2ax + a^2 = b^2$$

$$\therefore x^2 + 2ax + a^2 - b^2 = 0$$

If the roots of the equation are 2 and -1, then we know that

$$(x-2)(x+1) = 0$$

$$\therefore x^2 - x - 2 = 0$$

Equate the coefficients of the two versions of the equation

$$\therefore x^2 + 2ax + a^2 - b^2 = 0$$

$$\therefore x^2 - x - 2 = 0$$

For the *x* terms, $2a = -1$, which means that $a = -\frac{1}{2}$.

And for the constant terms, $a^2 - b^2 = -1$

$$\therefore b^2 = a^2 + 1 = \frac{1}{4} + 1 = \frac{9}{4}$$

$$\therefore b = \pm\frac{3}{2}$$

But $b > 0$ (absolute value), so $b = \frac{3}{2}$

So, the correct answer is B.

Question 5

How many integers satisfy the equation $(x^2 - 3x + 1)^{x+1} = 1$?

A. 1 B. 2 C. 3 D. 4

Solution

If $a^b = 1$, then we know that

1. *a* = 1

or 2. *b* = 0

or 3. *a* = -1 and *b* is even.

1. $x^2 - 3x + 1 = 1$

$$\therefore x(x-3) = 0$$

i.e., *x* = 0 or *x* = 3, which means there are two integer solutions.

2. $x + 1 = 0$

$\therefore x = -1$, which means that there is one integer solution.

3. $x^2 - 3x + 1 = -1$ and $x + 1$ is even.

$$x^2 - 3x + 2 = 0$$

$$\therefore (x-1)(x-2) = 0$$

$$\therefore x = 1 \text{ or } x = 2$$

and $x + 1$ is even when $x = 1$, which means there is one integer solution.

We can see that there are 2 + 1 + 1 = 4 integer solutions in total.

So, the correct answer is D.