NUMBER PATTERNS 10: QUIZ SOLUTIONS

Question 1

Which of the following is a divergent series?

A. $2+1+\frac{1}{2}+\cdots$ B. $1+2+4+\cdots$ C. $5+\frac{5}{4}+\frac{5}{16}+\cdots$ D. $9-3+1-\cdots$

Solution

A. $2+1+\frac{1}{2}+\cdots$ is a geometric series with $r=\frac{1}{2}$, so it converges.

B. $1+2+4+\cdots$ is a geometric series with $r = 2$, so it diverges.

C. $5+\frac{5}{4}+\frac{5}{16}+\cdots$ is a geometric series with $r=\frac{1}{4}$, so it converges.

D. $9-3+1-\cdots$ is a geometric series with $r=-\frac{1}{3}$, so it converges.

So, the correct answer is B.

Question 2

Which of the following is a convergent series?

A. $1+3+5+\cdots$ B. $1+1+2+3+\cdots$ C. $-3-6-9-\cdots$ D. $-2-1-\frac{1}{2}-\cdots$

Solution

A. $1+3+5+\cdots$ is an arithmetic series, terms are increasing, so it diverges.

B. $1+1+2+3+\cdots$ is the Fibonacci series, terms are increasing, so it diverges.

C. $-3-6-9-\cdots$ is an arithmetic series, terms are increasing negatively, so it diverges.

D. $-2-1-\frac{1}{2}-\cdots$ is a geometric series with $r=\frac{1}{2}$, so it converges.

So, the correct answer is D.

Question 3

An infinite geometric series is $(3-x)+(3-x)^2+(3-x)^3+\cdots(x\neq 3)$. For what value(s) of x will the series converge?

A. $2<x<4$ B. $-4<x<-2$ C. $x<2,\quad x>4$

D. $x<-4;\quad x>-2$

Solution

$a=3-x,\qquad r=3-x$

Converges if $-1<r<1$

$-1<3-x<1$

$-4<-x<-2$

$\therefore 2<x<4,\ (x\neq 3)$

So, the correct answer is A.

Question 4

What is the sum to infinity of the geometric series $18-3+\frac{1}{2}-\cdots$?

A. $-21\frac{3}{5}$ B. $-15\frac{3}{7}$ C. $15\frac{3}{7}$ D. $21\frac{3}{5}$

Solution

$$a=18, \quad r=-\tfrac{1}{6}$$

$$\therefore S_\infty=\frac{a}{1-r}=\frac{18}{1+\frac{1}{6}}$$

$$=\frac{108}{7}=15\tfrac{3}{7}$$

So, the correct answer is C.

Question 5

The sum to infinity of a geometric series is 15 and the common ratio is $\frac{2}{3}$. What are the first three terms?

A. 6; 4; $\frac{8}{3}$ B. 5; $\frac{10}{3}$; $\frac{20}{9}$ C. 4; $\frac{8}{3}$; $\frac{16}{9}$ D. 3; 2; $\frac{4}{3}$

Solution

$$S_\infty=15, \quad r=\tfrac{2}{3}$$

$$\therefore \frac{a}{1-\frac{2}{3}}=15$$

$$\therefore a=5=T_1$$

$$\therefore T_2=ar=5\times\tfrac{2}{3}=\tfrac{10}{3}$$

$$\therefore T_3=ar^2=\tfrac{10}{3}\times\tfrac{2}{3}=\tfrac{20}{9}$$

So, the correct answer is B.