

FUNCTIONS 7: QUIZ SOLUTIONS

Question 1

If $f(x) = \frac{1}{2}x^2$, what are its domain and range?

Solution

The domain of $f(x) = \frac{1}{2}x^2$ is $x \in R$, or $(-\infty; \infty)$ and the range is $y \in R, y \geq 0$, or $[0; \infty)$

Question 2

If $f(x) = 2x^2, x \leq 0$, what are the domain and range of its inverse?

Solution

The inverse of $f(x) = 2x^2, x \leq 0$ is $f^{-1}(x) = -\sqrt{\frac{x}{2}}, x \geq 0$. So, the domain of the inverse is $x \in R, x \geq 0$, or $[0; \infty)$ and the range is $y \in R, y \leq 0$, or $(-\infty; 0]$.

Question 3

If the inverse of a certain function is $f^{-1}(x) = -\sqrt{\frac{x}{3}}, x \geq 0$, then what is the certain function $f(x)$?

Solution

$$\text{Put } x = -\sqrt{\frac{y}{3}}$$

$$\therefore x^2 = \frac{y}{3}$$

$$\text{then } f(x) = 3x^2, x \leq 0.$$

Question 4

A function is defined by $f(x) = ax^2, a > 0, x \geq 0$. What is the equation of the line of symmetry of the function and its inverse?

Solution

The function and its inverse are symmetric about the line $x - y = 0$, or $y = x$.

Question 5

A function and its inverse are shown in the graph. If the function is $f(x) = 4x^2, x \geq 0$, what is its inverse?

Solution

$$f(x) = 4x^2, x \geq 0$$

$$y = 4x^2$$

$$\text{Put } x = 4y^2$$

$$\therefore y^2 = \frac{x}{4}$$

$$\text{then } f^{-1}(x) = \frac{1}{2}\sqrt{x}, x \geq 0.$$

