

FUNCTIONS 5: QUIZ SOLUTIONS

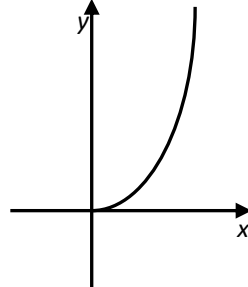
Question 1

If $f(x) = x^2$, $x \geq 0$, state whether its inverse is a one-to-one function, a many-to-one function, or not a function.

Solution

If $f(x) = x^2$, $x \geq 0$, then its graph looks like this:

So, it is a one-to-one function.



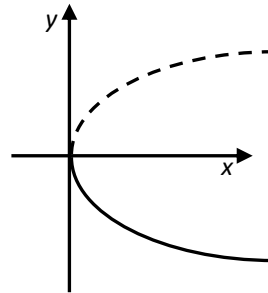
Question 2

Is $f(x) = \pm\sqrt{x}$ a one-to-one function, a many-to-one function, or not a function?

Solution

If $f(x) = \pm\sqrt{x}$, then its graph looks like this:

So, it is not a function.



Question 3

If $f(x) = 2x^2$, $x \geq 0$, then its inverse is a one-to-one function given by

- A. $f^{-1}(x) = 2\sqrt{x}$, $x \geq 0$ B. $f^{-1}(x) = \sqrt{\frac{x}{2}}$, $x \geq 0$
C. $f^{-1}(x) = \sqrt{2x}$, $x \geq 0$ D. $f^{-1}(x) = \frac{\sqrt{x}}{2}$, $x \geq 0$

Solution

If $f(x) = 2x^2$, $x \geq 0$, If $y = 2x^2$, put $x = 2y^2$

$$\therefore 2y^2 = x$$

$$\therefore y^2 = \frac{x}{2}$$

$$\therefore y = \pm\sqrt{\frac{x}{2}}$$

$$\text{But } x \geq 0, \text{ so } y = \sqrt{\frac{x}{2}}.$$

So, the correct answer is B. $f^{-1}(x) = \sqrt{\frac{x}{2}}$, $x \geq 0$.

Question 4

If $f(x) = x^2 + 4$, what are the restrictions on the domain of f so that its inverse is a one-to-one function?

Solution

$$f(x) = x^2 + 4$$

$$\text{Put } x = y^2 + 4$$

$$\therefore y = \pm\sqrt{x^2 - 4}$$

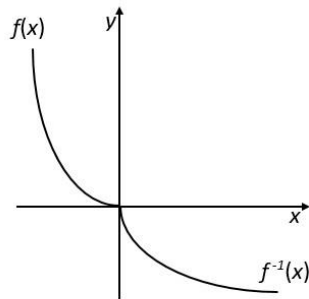
$$\therefore f(x)^{-1} = \sqrt{x^2 - 4} \text{ or } -\sqrt{x^2 - 4}$$

For the inverse to be a one-to-one function, the domain of f must be $x \leq 0$, or $x \geq 0$.

For the inverse, $x \geq 4$.

Question 5

A function and its inverse are shown in the following graph.



If the function is $f(x) = 3x^2$, $x \leq 0$, what is its inverse?

Solution

If $f(x) = 3x^2$, $x \leq 0$, then $f^{-1}(x) = -\sqrt{\frac{x}{3}}$, $x \geq 0$.