

FUNCTIONS 10: QUIZ SOLUTIONS

Question 1

A function is defined as $f(x) = -3x^2$. What are its domain and range?

Solutions

The domain and range of $f(x) = -3x^2$ are $(-\infty; \infty)$ and $(-\infty; 0]$.

Question 2

A function is defined as $f(x) = -\frac{x^2}{4}$, $x \geq 0$. What are the domain and range of its inverse?

Solution

$$\text{Put } x = -\frac{y^2}{4}$$

$$\therefore y^2 = -4x$$

$$\therefore y = \pm 2\sqrt{-x}$$

So, the inverse of $f(x) = -\frac{x^2}{4}$, $x \geq 0$ is $f^{-1}(x) = -2\sqrt{-x}$, $x \leq 0$.

The domain of $f^{-1}(x)$ is $(-\infty; 0]$ and the range is $[0; \infty)$.

Question 3

If the inverse of a certain function is $f^{-1}(x) = -\frac{1}{3}\sqrt{-x}$, $x \leq 0$, what is the function $f(x)$?

Solution

$$\text{Put } x = -\frac{1}{3}\sqrt{-y}$$

$$\therefore 3x = -\sqrt{-y}$$

$$\therefore 9x^2 = -y$$

So, $f(x) = -9x^2$, $x \leq 0$.

Question 4

If a function is defined by $f(x) = ax^2$, $a > 0$, $x \leq 0$, then the function and its inverse are symmetric about which line?

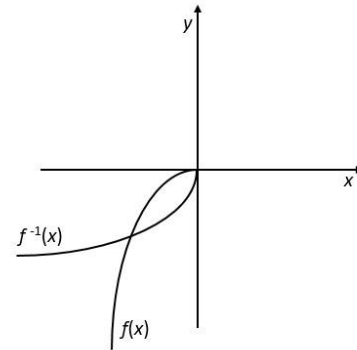
Solution

The line of symmetry is $y = x$, or $x - y = 0$.

Question 5

Which function and its inverse are shown by the following graph?

- A. $f(x) = 16x^2$, $x \leq 0$; $f^{-1}(x) = \frac{1}{4}\sqrt{x}$, $x \leq 0$
- B. $f(x) = 4x^2$, $x \leq 0$; $f^{-1}(x) = \frac{1}{4}\sqrt{x}$, $x \leq 0$
- C. $f(x) = -16x^2$, $x \leq 0$; $f^{-1}(x) = -\frac{1}{4}\sqrt{-x}$, $x \leq 0$
- D. $f(x) = -4x^2$, $x \leq 0$; $f^{-1}(x) = -\frac{1}{4}\sqrt{-x}$, $x \leq 0$

**Solution**

- C. $f(x) = -16x^2$, $x \leq 0$; $f^{-1}(x) = -\frac{1}{4}\sqrt{-x}$, $x \leq 0$