PROBABILITY 3: QUIZ SOLUTIONS

Question 1

What is the probability that of all possible 3-digit numbers, excluding those whose first digit is zero, the first digit will be a 9?

A. $\frac{1}{10}$ B. $\frac{1}{9}$ C. $\frac{8}{9}$ D. $\frac{9}{10}$

Solution

There are 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Number of *possible* ways:

1st digit may not be zero, so 9 ways
2nd digit may be any digit, so 10 ways
3rd digit may be any digit, so 10 ways

So, number of *possible* ways = 9 x 10 x 10

Number of *favourable* ways for first digit to be a 9, i.e., 9XX

1st digit must be a 9, so only 1 way
2nd digit may be any digit, so 10 ways
3rd digit may be any digit, so 10 ways

So, number of *favourable* ways = 1 x 10 x 10

$$\therefore \text{P(9XX)} = \frac{1\times 10\times 10}{9\times 10\times 10} = \frac{1}{9}$$

So, the correct answer is B.

Question 2

What is the probability that of all possible 3-digit numbers, excluding those whose first digit is zero, there will be only one 9?

A. $\frac{1}{10}$ B. $\frac{1}{9}$ C. $\frac{1}{8}$ D. $\frac{1}{4}$

Solution

Number of *possible* ways = 9 x 10 x 10 = 900

Number of *favourable* ways for only one 9, i.e., 9XX or X9X or XX9

For 9XX, number of ways = 1 x 9 x 9
OR For X9X, number of ways = 8 x 1 x 9 (no zero)
OR For XX9, number of ways = 8 x 9 x 1 (no zero)

So, number of *favourable* ways = (1 x 9 x 9) + (8 x 1 x 9) + (8 x 9 x 1)
= 81 + 72 + 72
= 225

$$\therefore \text{P(only one 9)} = \frac{225}{900} = \frac{1}{4}$$

So, the correct answer is D.

Question 3

What is the probability that of all possible 3-digit numbers, excluding those whose first digit is zero, there will be exactly two 9s?

A. $\frac{1}{1000}$ B. $\frac{1}{100}$ C. $\frac{13}{450}$ D. $\frac{1}{3}$

Solution

Number of *possible* ways = 9 x 10 x 10 = 900

Number of *favourable* ways for exactly two 9s, i.e., 99X or X99 or 9X9

For 99X, number of ways = 1 x 1 x 9

OR For X99, number of ways = 8 x 1 x 1 (no zero)

OR For 9X9, number of ways = 1 x 9 x 1

So, number of *favourable* ways = (1 x 1 x 9) + (8 x 1 x 1) + (1 x 9 x 1)

= 9 + 8 + 9

= 26

$$\therefore \text{P(exactly two 9s)} = \frac{26}{900} = \frac{13}{450}$$

So, the correct answer is C.

Question 4

What is the probability that of all possible 3-digit numbers, excluding those whose first digit is zero, at least one 9 will occur?

A. $\frac{1}{10}$ B. $\frac{1}{9}$ C. $\frac{1}{4}$ D. $\frac{7}{25}$

Solution

Number of *possible* ways = 9 x 10 x 10 = 900

Number of *favourable* ways for at least one 9

i.e., (9XX or X9X or XX9) or (99X or X99 or 9X9) or (999)

For (9XX or X9X or XX9), number of ways = (1 x 9 x 9) + (8 x 1 x 9) + (8 x 9 x 1)

= 81 + 72 +72

= 225

For (99X or X99 or 9X9), number of ways = (1 x 1 x 9) + (8 x 1 x 1) + (1 x 9 x 1)

= 9 + 8 +9

= 26

For (999), number of ways = 1 x 1 x 1 =1

Total number of favourable ways = 225 + 26 + 1

= 252

$$\therefore \text{P(at least one 9)} = \frac{252}{900} = \frac{7}{25}$$

So, the correct answer is D.

An alternative (preferred/easier?) method is to use the fact that

P(at least one 9) = 1 – P(no 9s) i.e., XXX

Number of favourable ways of no 9s = 8 x 9 x 9 = 648

$$\therefore \text{P(at least one 9)} = 1 - \text{P(no 9s)} = 1 - \frac{648}{900} = \frac{252}{900} = \frac{7}{25}$$

Question 5

What is the probability that there will be two consecutive 9s?

A. $\frac{1}{50}$ B. $\frac{17}{900}$ C. $\frac{9}{500}$ D. $\frac{4}{225}$

Solution

Number of *possible* ways = 9 x 10 x 10 = 900

Number of *favourable* ways for two consecutive 9s

i.e., 99X or X99

For (99X or X99), number of ways = (1 x 1 x 9) + (8 x 1 x 1)

= 9 + 8 = 17

$$\therefore \text{P(two consecutive 9s)} = \frac{17}{900}$$

So, the correct answer is B.