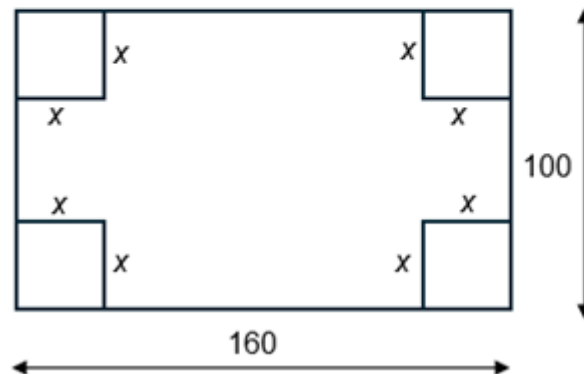


CALCULUS 15: QUIZ SOLUTIONS

A piece of cardboard 160 mm by 100 mm has four squares of side x mm cut from the corners, as shown in the diagram. The cardboard is then folded to form an open box of depth x mm. The task is to find the value of x that will make the volume of the box a maximum.



Question 1

What is the volume, V , of the box in terms of x ?

- A. $V = 2x(x - 25)(x - 40)$ B. $V = 2x(x - 50)(x - 80)$
C. $V = 4x(50 - x)(80 - x)$ D. $V = 4x(25 - x)(40 - x)$

Solution

$$\text{Length} = 160 - 2x$$

$$\text{Width} = 100 - 2x$$

$$\begin{aligned}\text{Volume is } V &= x(160 - 2x)(100 - 2x) \\ &= 4x(80 - x)(50 - x)\end{aligned}$$

So, the correct answer is C.

Question 2

What is the first derivative $\frac{dV}{dx}$?

- A. $\frac{dV}{dx} = 6x^2 - 260x + 8000$ B. $\frac{dV}{dx} = 12x^2 - 1040x + 16000$
C. $\frac{dV}{dx} = 12x^2 - 520x + 4000$ D. $\frac{dV}{dx} = 6x^2 - 260x + 2000$

Solution

$$V = 4x(50 - x)(80 - x) = 4x^3 - 520x^2 + 16000x$$

$$\therefore \frac{dV}{dx} = 12x^2 - 1040x + 16000$$

So, the correct answer is B.

Question 3

When $\frac{dV}{dx} = 0$, what are the values of x ?

- A. 10 or $\frac{100}{3}$ B. 50 or $\frac{20}{3}$ C. 51,1 or 7,7 D. 20 or $\frac{200}{3}$

Solution

$$12x^2 - 1040x + 16000 = 0$$

$$\therefore 3x^2 - 260x + 4000 = 0$$

$$\therefore (3x - 200)(x - 20) = 0$$

$$\therefore x = \frac{200}{3} \text{ or } x = 20$$

So, the correct answer is D.

Question 4

Which value of x will give the maximum volume of the box?

- A. 7,7 B. 10 C. 20 D. 50

Solution

$$\frac{dV}{dx} = 12x^2 - 1040x + 16000$$

$$\therefore \frac{d^2V}{dx^2} = 24x - 1040$$

$$\text{When } x = \frac{200}{3}, \frac{d^2V}{dx^2} = 24 \times \frac{200}{3} - 1040 = 1600 - 1040 > 0 \Rightarrow \text{min}$$

$$\text{When } x = 20, \frac{d^2V}{dx^2} = 24 \times 20 - 1040 = 480 - 1040 < 0 \Rightarrow \text{max}$$

So, the correct answer is C.

Question 5

What is the maximum volume of the box?

- A. 18 000 mm³ B. 25 000 mm³
C. 47 097,7 mm³ D. 144 000 mm³

Solution

$$V = 4x(50 - x)(80 - x)$$

$$V_{\max} = 4(20)(50 - 20)(80 - 20)$$

$$= 80 \times 30 \times 60$$

$$= 144\,000$$

So, the correct answer is D.