CALCULUS 10: QUIZ SOLUTIONS

Question 1

If $f(x)=4x^2+\frac{27}{x}$, what is $f''(x)$?

A. $8+\frac{54}{x^3}$ B. $8x+\frac{27}{x^2}$ C. $8-\frac{54}{x^3}$ D. $8x-\frac{27}{x^2}$

Solution

$$f(x)=4x^2+\frac{27}{x}=4x^2+27x^{-1}$$
$$\therefore f'(x)=2\times 4x^{2-1}+(-1)\times 27x^{-1-1}$$
$$=8x-27x^{-2}$$
$$\therefore f''(x)=8-(-2)\times 27x^{-2-1}$$
$$=8+54x^{-3}$$
$$=8+\frac{54}{x^3}$$

So, the correct answer is A.

Question 2

If a function $f(x)$ is concave up, what is true of the second derivative $f''(x)$?

A. $f''(x)<0$ B. $f''(x)=0$ C. $f''(x)>0$

Solution

If $f''(x)<0$, then the function is concave down.
If $f''(x)>0$, then the function is concave up.
If $f''(x)=0$, then there is a point of inflection.

So, the correct answer is C.

Question 3

If a function $f(x)$ is concave down, what is true of the second derivative $f''(x)$?

A. $f''(x)<0$ B. $f''(x)=0$ C. $f''(x)>0$

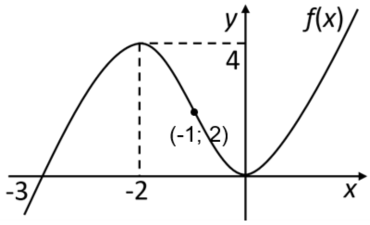
Solution

If $f''(x)<0$, then the function is concave down.
If $f''(x)>0$, then the function is concave up.
If $f''(x)=0$, then there is a point of inflection.

So, the correct answer is A.

Question 4

The graph of $f(x)$ is shown in the diagram. For which values of x is $f(x)$ concave down?



A. $(-3; 0)$ B. $(-2; 4)$ C. $(-1; \infty)$ D. $(-\infty; -1)$

Solution

From the graph, x-intercepts are -3 and 0 (repeated root)

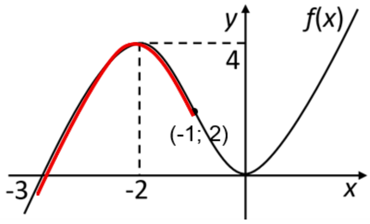
$$\therefore x^2(x+3)=0$$

$$\text{i.e., } f(x)=x^2(x+3)=x^3+3x^2$$

$$f'(x)=3x^2+6x$$

$$\text{and } f''(x)=6x+6$$

When $f''(x)=0$, $x=-1$, which indicates a point of inflection

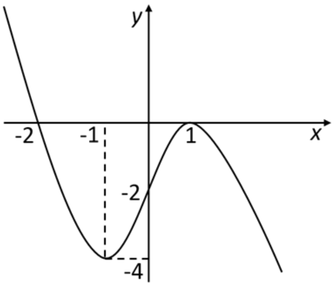


This shows that $f(x)$ is concave down from $-\infty$ to -1.

So, the correct answer is D.

Question 5

The graph of $f(x)$ is shown in the diagram. For which values of x is $f''(x) > 0$?



A. $(-\infty; 0)$ B. $(0; \infty)$ C. $(-2; -1)$ D. $(-4; \infty)$

Solution

From the graph, x-intercepts are -2 and 1 (repeated root)

$$\therefore (x+2)(x-1)^2 = 0$$

i.e., $f(x) = -(x+2)(x-1)^2 = -x^3 + 3x - 2$ (Note minus sign)

$$f'(x) = -3x^2 + 3x$$

and $f''(x) = -6x$

When $f''(x) = 0$, which indicates a point of inflection, $x = 0$

$$\therefore f''(x) > 0 \text{ for } x < 0$$

So, the correct answer is A.